

THE PROBABILISTIC ANALYSIS OF RSSI-BASED DISTANCE ESTIMATORS IN WIRELESS SENSOR NETWORK

Chuanzhi Zang^{1,2} Wei Liang¹ Haibin Yu^{*1}

1 Shenyang Institute of Automation, Chinese Academy of Sciences,
Shenyang 110016, China

2 Graduate School of the Chinese Academy of Sciences,
Beijing 100049, China

E-mail: {[zangcz](mailto:zangcz@sia.cn), [weiliang](mailto:weiliang@sia.cn), [yhb](mailto:yhb@sia.cn)}@sia.cn

ABSTRACT

It is well known that localization plays an important role in wireless sensor network applications. There are two categories of localization approaches, such as range-based and range-free. RSSI(Received Signal Strength Indicator)-based method is in the first category. Three RSSI-based distance estimators, Biased Estimator (BE), Unbiased Estimator (UE) and Maximal Likelihood Estimator (MLE), are presented to calculate the distance. The biased estimator is the existing method which is used extensively in the literatures, while the unbiased estimator and maximal likelihood estimator are two new estimators developed in this paper. The probabilistic analysis of those estimators is done to compare the performance among them. The probabilistic analysis and simulations show that under some condition we should choose unbiased estimator and under some other condition we should choose MLE.

1. INTRODUCTION

Recent technological advances in distributed embedded systems have prompted significant research efforts in both the industry and the academia. Among such systems, wireless sensor networks are particularly noteworthy due to their potentially numerous, economically attractive applications and their ability to bridge the interface between the user and the physical world^[1]. There are many challenges in wireless sensor network.

Emerging applications of wireless sensor networks will depend on automatic and accurate location of thousands of sensors. In environmental sensing applications such as water quality monitoring, precision agriculture, and indoor air quality monitoring, “sensing data without knowing the sensor location is meaningless”^[2]. In addition, by helping reduce configuration requirements and device cost, estimation of sensor location in wireless sensor networks may enable applications such as inventory management, intrusion detection^[3], target tracking, traffic monitoring, and locating emergency workers in buildings. Finally, knowing the relative locations of sensors allows use of location-based addressing and routing protocols, which can improve network robustness and energy-efficiency^[4].

So sensor network localization plays an important role in wireless sensor network applications. Nodes could be equipped with a global positioning system

(GPS) to provide them with absolute positions, but this is currently an infeasible solution due to the cost. The existing work on sensor network localization can be categorized into range-based and range-free approaches. Ranged-based approaches are commonly used as a method to obtain the fine-grained location information of nodes^[5]. Generally speaking, the range-based location discovery approaches consist of two basic phased: (1) distance (or angle) estimation and (2) distance (or angle) combining. The approaches, Time of Arrival (TOA), Time Difference of Arrival (TDOA), Angle of Arrival (AOA) and Received Signal Strength Indicator (RSSI), are typical range-based approaches. As an alternate category of range-based localization approaches, range-free localization methods have been proposed to obtain coarse-grained location estimates in an acceptable limit. In this category, there are three major schemes: Centroid, DV-HOP and Approximate Point-In-Triangulation (APIT)^[6].

In this paper we focus on the RSSI-based localization approach. The existing RSSI based localization algorithms use RSSI in two different ways: direct mapping between RSSI and distance (location) and indirect mapping. Ours is in the category of direct mapping. Though RSSI is used extensively in the literatures^[7-10], few researchers analyzes the estimators probabilistically. The log-distance path loss model^[7-10] (see equation (1)) is used to map the RSSI to distance. According the log-distance path loss model, Kuo^[7] designs a distance estimator (We call is as biased estimator) which is same as equation (4). Kuo tries to analyze the estimator probabilistically, but we will see that the analysis isn't correct. Ahmed^[8] uses RSSI model to calculate the detectable probability of a target for a sensor. The distance estimator used in [9] is similar to equation (4), however, it also considers the influence of the building wall. The distance estimator used in [10] is the same as Kuo's^[7].

Except for the biased estimator, two new RSSI-based distance estimators, unbiased estimator and maximal likelihood estimator, are developed in this paper. The probabilistic analysis of those estimators is done to compare the performance of each estimator. The analysis shows why error between the true distance and the biased estimator is very big. The comparison among the estimators

shows that each estimator suits to some special conditions.

The remainder of this paper is organized as follows. In section 2, the log distance path model and the three distance estimators are addressed. Section 3 compares the performance of these distance estimators. The simulations are given in section 4. Section 5 concludes the paper.

2. THREE ESTIMATORS

2.1. Log-distance path loss model

Each sensor estimates the distance based on the RSSI. There are three basic propagation models^[5] about the RSSI: the free space model, two-ray ground reflection model and the log-normal shadowing model. The free space model and the two-ray model predict the received power as a deterministic function of distance. They both represent the communication range as an ideal circle. In reality, the received power at certain distance is a random variable due to multi-path fading effects. The log-normal shadowing model is more general and widely-used. The shadowing model extends the ideal circle model to a richer statistic model. From log-normal shadowing model, we can get the log-distance path loss model as

$$\begin{aligned} PL(d) &= \overline{PL}(d) + X_\sigma \\ &= \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right) + X_\sigma \end{aligned} \quad (1)$$

where n is a path loss exponent which indicates the decreasing rate of signal strength in an environment, d_0 is the reference distance which is close to the transmitter, d is the distance between the transmitter and receiver, $\overline{PL}(d_0)$ is the mean path loss at reference distance d_0 (in dB), $\overline{PL}(d)$ is the mean path loss at reference distance d (in dB), X_σ is zero-mean Gaussian distributed random variable (in dB) with σ variance and $PL(d)$ is the path loss at distance d . So $PL(d)$ is a Gaussian normal distribution. Note that equation (1) considers the factors in the surrounding environment, such as interference, multi-path and fading. Furthermore, from equation (1), we can get the function relationship between $\overline{PL}(d_0)$ and $\overline{PL}(d)$ as

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n \log\left(\frac{d}{d_0}\right) \quad (2)$$

2.2. Three distance estimators

Because $PL(d)$ is a random variable, how to design a distance estimator is a very important thing. In this subsection we show three different distance estimators.

2.2.1. Biased estimator

According to equation (2), if we can get $\overline{PL}(d)$, the true distance d can be easily estimated as

$$d = d_0 10^{\frac{\overline{PL}(d) - \overline{PL}(d_0)}{10n}} \quad (3)$$

The error between the estimated distance and true distance is 0. However, it is just an ideal estimator. In reality, we can only get the sample of $PL(d)$, and the corresponding estimator is

$$d_b = d_0 10^{\frac{PL(d) - \overline{PL}(d_0)}{10n}} = d 10^{\frac{X_\sigma}{10n}} \quad (4)$$

where d_b is the biased estimator of d . Equation (4) is used to estimate the distance extensively in the literature^[7-10]. We will see equation (4) can result in big error between estimation value d_b and true value d under some special conditions in true physical world.

Because X_σ is a random variable and d_b is function of X_σ , d_b is a random variable. The Probability Density Function (PDF) of d_b is

$$p_{d_b}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(10n \log(x/d))^2}{2\sigma^2}\right) \frac{10n}{x \ln 10}, x > 0 \quad (5)$$

Note that the PDF of Kuo's^[7] is as following:

$$p_{Kuo}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(10n \log(x/d))^2}{2\sigma^2}\right) \quad (6)$$

which is not a PDF of a random variable. So Kuo's probabilistic analysis based on (6) is wrong.

Let $n=2$ and $d=10$, figure 1 shows the PDFs of d_b for some special values of σ . By intuition, when parameter σ increases, the expectation value of d_b decreases.

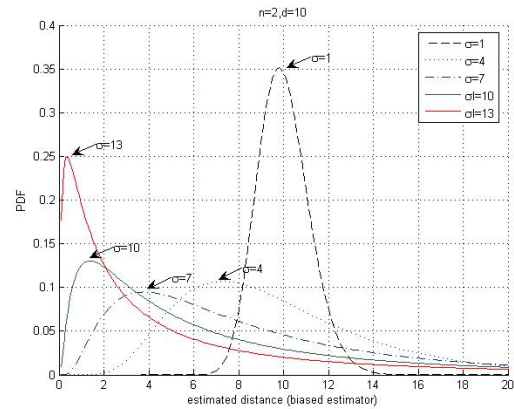


Figure1. The PDF of biased estimator

In fact, it is true. The expectation value and variance value of d_b are

$$E(d_b) = d \exp\left(\frac{\sigma^2 \ln^2 10}{200n^2}\right) \quad (7)$$

$$D(d_b) = d^2 \left(\exp\left(\frac{\sigma^2 \ln^2 10}{50n^2}\right) - \exp\left(\frac{\sigma^2 \ln^2 10}{100n^2}\right) \right) \quad (8)$$

which are the functions of σ . According to equation (7), one can conclude that d_b is a biased estimator of d . This means that the average value of several sample values of d_b has a distance to the true value d . Then what is the unbiased estimator of d ?

2.2.2. Unbiased estimator

Let us construct a function of d_b or X_σ as following

$$\begin{aligned} d_u &= d_b \exp\left(-\frac{\sigma^2 \ln^2 10}{200n^2}\right) \\ &= d_0 \exp\left(\frac{(PL(d) - PL(d_0)) \ln 10}{10n} - \frac{\sigma^2 \ln^2 10}{200n^2}\right) \\ &= d \exp\left(\frac{X_\sigma \ln 10}{10n} - \frac{\sigma^2 \ln^2 10}{200n^2}\right) \end{aligned} \quad (9)$$

So d_u is a random variable and the corresponding PDF is

$$p_{d_u}(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(10n \log(y/d) + \frac{\sigma^2 \ln 10}{20n}\right)^2}{2\sigma^2}\right) \frac{10n}{y \ln 10}, y > 0 \quad (10)$$

One can easily proof that d_u is the unbiased estimator of true distance d . Here we list the expectation value and variance value of d_u :

$$E(d_u) = d \quad (11)$$

$$D(d_u) = d^2 \left(\exp\left(\frac{\sigma^2 \ln^2 10}{100n^2}\right) - 1 \right) \quad (12)$$

Let $n=2$ and $d=10$, figure 2 shows the PDFs of d_u for some special values of σ .

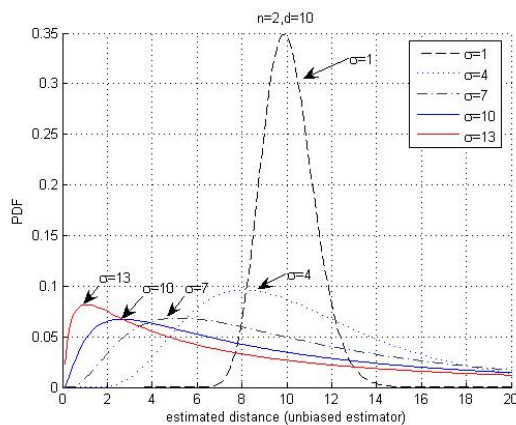


Figure 2. The PDF of unbiased estimator
From figure 1 and figure 2, one can conclude that biased estimator and unbiased estimator don't get the maximum value of PDF at d , which means that the estimated distance can't approach the true distance d with the maximum probability.

2.2.3. Maximum likelihood estimator

In this subsection we construct the maximum likelihood estimator, the PDF of which can get the maximum value at true distance d . Let the derivative of biased estimator's PDF equals 0, then we get

$$x = d \exp\left(-\frac{\sigma^2 \ln^2 10}{100n^2}\right) \quad (13)$$

So we construct the MLE as

$$\begin{aligned} d_M &= d_b \exp\left(\frac{\sigma^2 \ln^2 10}{100n^2}\right) \\ &= d_0 \exp\left(\frac{(L - PL(d_0)) \ln 10}{10n} + \frac{\sigma^2 \ln^2 10}{100n^2}\right) \\ &= d \exp\left(\frac{X_\sigma \ln 10}{10n} + \frac{\sigma^2 \ln^2 10}{100n^2}\right) \end{aligned} \quad (14)$$

The proof is very easy, so we don't show the proof in this paper. The corresponding PDF is

$$p_{d_M}(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(10n \log(z/d) - \frac{\sigma^2 \ln 10}{10n}\right)^2}{2\sigma^2}\right) \frac{10n}{z \ln 10}, z > 0 \quad (15)$$

Similar to the biased and unbiased estimators, we also calculate the expectation and variance of d_M as following

$$E(d_M) = d \exp\left(\frac{3\sigma^2 \ln^2 10}{200n^2}\right) \quad (16)$$

$$D(d_M) = d^2 \left(\exp\left(\frac{\sigma^2 \ln^2 10}{25n^2}\right) - \exp\left(\frac{3\sigma^2 \ln^2 10}{100n^2}\right) \right) \quad (17)$$

Let $n=2$ and $d=10$, figure 3 shows the PDFs of d_M for some special values of σ .

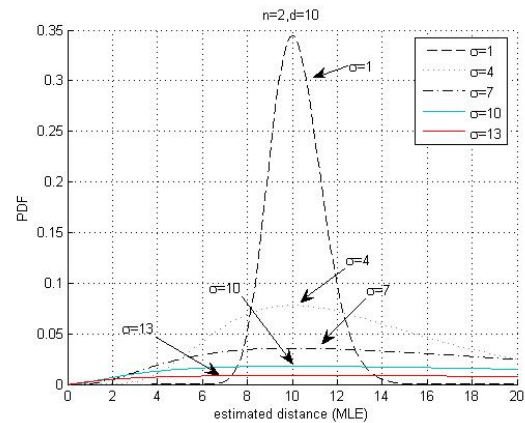


Figure 3. The PDF of MLE
The PDFs plotted in figure 3 get their maximum value at true distance $d=10$.

3. THE COMPARISON OF THE THREE ESTIMATORS

In the above section, we show the three estimators and analyze the expectation and variance for each estimator. In this section we compare the performance of those estimators.

In the remains of this paper, we use d_e to denote one of the three estimators. In order to show how the parameter σ influence the expectation of the estimators, we plot $E(d_e)/d$ in Figure 4 ($n=2$).

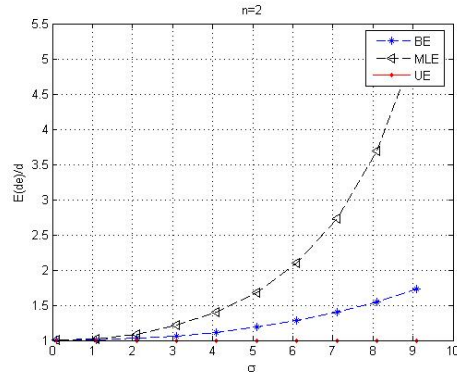


Figure 4. The relationship between $E(d_e)/d$ and σ . As σ increases, the $E(d_e)/d$ of each estimators increases and is always larger than 1. Furthermore, the $E(d_e)/d$ of MLE is always larger than all other estimators and the $E(d_e)/d$ of UE has no relationship with σ . For biased estimator which is used extensively in the literature, the value of $E(d_e)/d$ is about 1.5 at $\sigma=8$. Thus the error between the estimated distance and true distance is too big to be used.

Let $n=2$ and $d=10$, we plot all variances of the estimators in figure 5. Similar to $E(d_e)/d$, as σ increases, the variance of each estimators increases. The variance of MLE is always larger than other estimators, while the variance of UE is always less than other estimators.

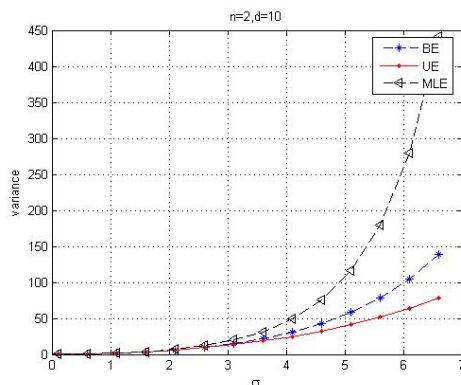


Figure5. The variance of three estimators

From figure 4 and figure 5, we can see that the unbiased estimator is always better than biased and MLE estimators.

4. SIMULATIONS

In this section, we compare the three estimators and Kuo's estimator (see equation (6)) for different values of parameter σ by simulations and show how multi-sensors coordinately sense a target.

We use MATLAB to simulate a 50×50 area. m sensors, denoted by $s_i, i=1, \dots, m$, are deployed in the area. Suppose the target location is l . Given a point e in the region, for single sensor s_i and biased estimator, the possibility that the sensor s_i estimates the location l is at e is denoted by:

$$poss_{s_i}(e) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left(-\frac{\left(10n \log \left(\frac{dist(s_i, e)}{dist(s_i, l)} \right) \right)^2}{2\sigma^2} \right) \frac{10n}{dist(s_i, e) \ln 10} \quad (18)$$

where $dist(s_i, e)$ is the Euclidean distance between s_i and e and $dist(s_i, l)$ is the Euclidean distance between s_i and l . For multi-sensor, we use the corresponding joint PDF to denote the possibility (see equation (18)). For other estimators, we can get the corresponding possibilities.

$$poss(e) = \prod_{i=1}^m poss_{s_i}(e) \quad (19)$$

We first place a sensor at location (15, 25) and a target at (25, 35). Figure 6 and figure 7 show the possibility distribution of the estimated location of the target for different estimators and values of σ . For $\sigma=5$, figure 6 plot the possibility distributions. The high possibility area of Kuo's estimator and MLE is greater than that of biased and unbiased estimators, which means that the domain of the estimated distance with high probability of Kuo's estimator and MLE is bigger than that of biased and unbiased estimators, that is to say that the error or variance of Kuo's estimator and MLE is bigger than that of biased and unbiased estimators which is consistent with figure 5. While the high possibility area of biased and unbiased estimators is less that of Kuo's estimator and MLE, the true location of the target doesn't stay at the high possibility area.

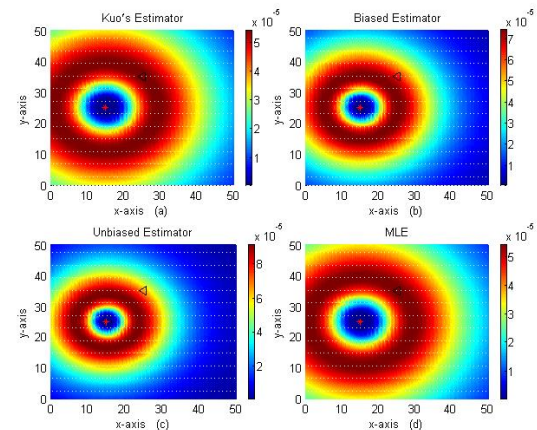


Figure 6. Possibility distribution for one sensor and $\sigma=5$

For $\sigma=1$, figure 7 plot the possibility distributions. For each estimator, the high possibility area is very small and the difference among them is also very small. Different from figure 6, the true location of the target stays at the high possibility area. We can

conclude that when σ turns small, the estimated distance converges to the true distance.

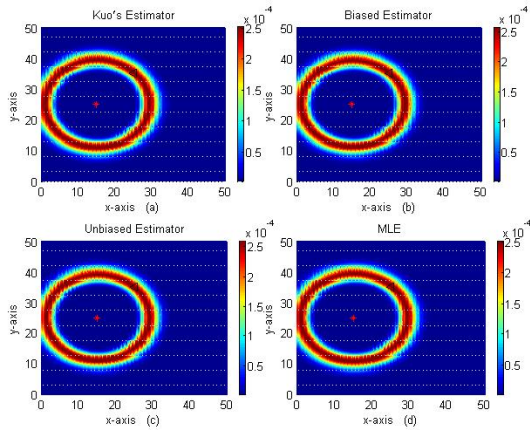


Figure 7. Possibility distribution for one sensor and $\sigma=1$

We place the second sensor at (35, 25) and plot the possibility distributions in figure 8 and figure 9. Similar to figure 6, when $\sigma=5$, the high possibility area of Kuo's estimator and MLE is larger than that of biased and unbiased estimator. However, for each estimator, the high possibility area is smaller than that of figure 6 and is divided into two areas. The target stays at one of the areas. When $\sigma=1$, the difference among those estimators is very small and the target stay at one of the two *points*.

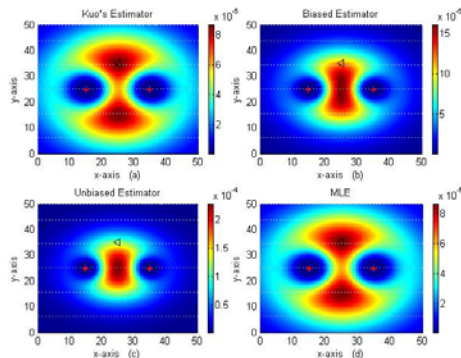


Figure 8. Possibility distribution for two sensors and $\sigma=5$

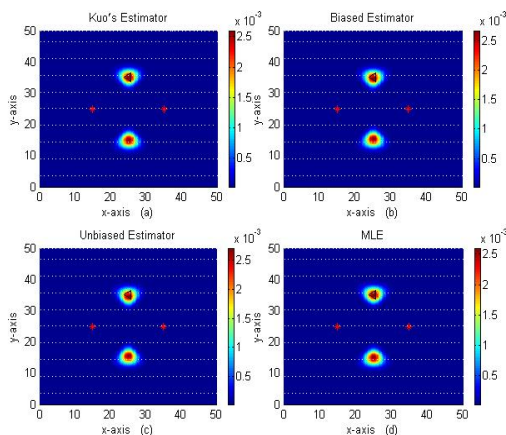


Figure 9. Possibility distribution for two sensors and $\sigma=1$

Finally, we place the third sensor at (40, 45) and plot the possibility distributions in figure 10 and figure 11. When $\sigma=5$, the high possibility area of each estimator is reduced again (see figure 10). The coordination sensing of the three sensors reduces the high possibility area, which make the estimated error turn small. When $\sigma=1$, the difference among those estimators is very small and the target stay at only one *point*. Those sensors *capture* the target.

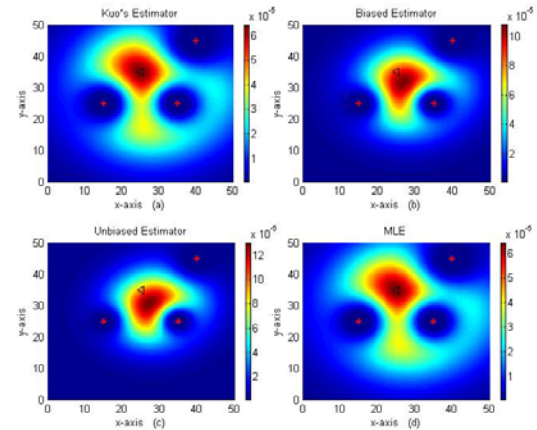


Figure 10. Possibility distribution for three sensors and $\sigma=5$

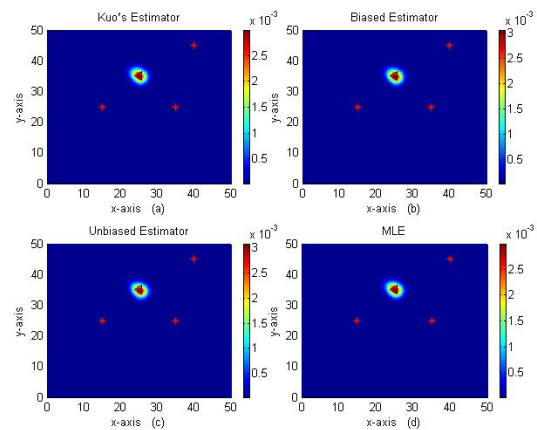


Figure 11. Possibility distribution for three sensors and $\sigma=1$

According to these simulations, when σ is big, the high possibility area of each estimator is also big and the high possibility area of Kuo's estimator and MLE is larger than that of biased and unbiased estimators. For each estimator, the larger the high possibility area is, the bigger the estimated error or variance is. When σ is small, the estimated distance converges to the true distance probabilistically. When more sensors sense the target, the high possibility area turn smaller and three sensors can *capture* the target. Note that though the figure of Kuo's estimator plotted by equation (6) is very similar to MLE in all the simulations, it isn't described the estimator correctly. In fact, the figure of biased estimator is the correct description.

5. CONCLUSION

Based on the RSSI, three estimators, biased estimator, unbiased estimator and MLE, are analyzed probabilistically. The analysis shows that when σ is big and we only keep the expectation and variance in brain, we should choose the unbiased estimator to estimate the distance. If we keep the detectable possibility in mind, we should choose MLE. The performance of biased is between the unbiased estimator and MLE. On the other side, when σ is small, the difference among the estimators is also small. We can choose anyone of them to estimate the distance.

6. ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China (No. 60374072, 60434030).

7. REFERENCES

- [1] D. Estrin, R. Govindan, J. Heidemann, and S. Kumar, "Next Century Challenges: Scalable Coordination in Sensor Networks," *In Proc. of MobiCOM*, pp 263–270, 1999.
- [2] J. M. Rabaey, M. J. Ammer, J. L. da Silva, Jr., D. Patel, and S. Roundy, "Picoradio Supports Ad Hoc Ultra-low Power Wireless Networking," *IEEE Computer*, 33(7), pp 42–48, July 2000.
- [3] R. L. Moses, D. Krishnamurthy, and R. Patterson, "An Auto-calibration Method for Unattended Ground Sensors," *In ICASSP*, vol. 3, pp 2941–2944, May 2002.
- [4] Y. B. Ko and N. H. Vaidya, "Location-aided Routing (LAR) for Mobile Ad-hoc Networks," *In ACM / IEEE MOBIKOM '98*, pp307–321, Oct. 1998.
- [5] X. F. Shen, Z. Wang, R. Z. Lin, Y. X. Sun, "Connectivity and RSSI Based Localization Scheme for Wireless Sensor Networks," *Lecture Notes on Computer Science, ICIC 2005, Hefei, China*, Aug.2005
- [6] N. Bulusu, J. Heidemann, D. Estrin, "Gps-less Low Cost Outdoor Localization for Very Small Devices," *IEEE Personal Communications Magazine*. 7, pp28–34, 2000.
- [7] S. P. Kuo, Y. C. Tseng, F. J. Wu, and C. Y. Lin, "A Probabilistic Signal-Strength-Based Evaluation Methodology for Sensor Network Deployment," *Int. J. of Ad Hoc and Ubiquitous Computing*, vol. 1, Nos. 1/2, pp.3–12, 2005.
- [8] N. Ahmed, S. S. Kanhere, S. Jha, "Probabilistic Coverage in Wireless Sensor Networks," *The IEEE Conference on Local Computer Networks*, (WLN 2005). 30th Anniversary. pp672–681, 2005.
- [9] P. Bahl, V. Padmanabhan, "RADAR: An In-Building RF-based User Location and Tracking System," *In: Proceedings of INFOCOM 2000*, vol. 2, pp775–784, 2000.
- [10] N. Patwari, R. J. O'Dea, and Y. Wang, "Relative Location in Wireless Networks," *In Proc. IEEE VTC*, vol. 2, pp. 1149–1153, May 2001.

This Paper from

International Conference on Intelligent Computing 2007, August 21–24, 2007 in Qingdao Shandong Province, China.

CIMS Papers' World

Here scientific papers on Computer Integrated Manufacturing System(CIMS) and other fields!for you FREELY!

<http://www.cimspaper.com>